χ, μ,
$\mu = 0$, ($u - \mu u^2$
fi)

, . . $r = 0, \mu = 0,$ (1.1) K 15 ,
1970. K - -
- fi / fi , . A fi / fi -
- 1-D 8,25,37 , - 2-D: $\|u_0\|_{L^1} < 4\pi/\chi$, -
, , $\|u_0\|_{L^1} > 4\pi/\chi$,
fi , . 11,3,22,21,26 , - $\geq$ 3-D 31,33 . 1,12
K .
M 20 ,
, , . F ,
K . O , - fl
K . I , $n = 1, 2,$
$\mu > 0$ - (1.1) - -
8,25,24,38 . 2-D fi (DE (1.1)
$0 = \Delta v - v + u$) 5 , , $n \geq 3$,
fi fi (1.1) $\mu > \frac{(n-2)}{n}\chi$ 30 . N ,
$\mu \geq \frac{n-2}{n}\chi$ 10,14,40 . M , - , u
$\frac{r}{\mu}$ 18,35 . C
(1.1), Ω , fi μ
μ_0 32 . A $\mu > \theta_0\chi$
θ_0 41 . A 3-D -fl
, (1.1) $\chi = 1$,

$$E(\chi,\mu)=e^{\frac{\chi C_{GN}^2}{2\ \{1,\frac{2}{\chi}\}}\left[\frac{(r+3)}{\mu}\|u_0\|_{L^1(\Omega)}+\frac{(r+1)^3}{4\mu^2}|\Omega|+\|\nabla v_0\|^2_{L^2(\Omega)}+\frac{(r+2)^2}{2\mu^2}|\Omega|\right]}, \tag{1.5}$$

C_{GN} G N L 2.1 :

$$C_{GN}:=2C_{GN}^2(4,2,1,\Omega). \tag{1.6}$$

, 1.1 $\|u(t)\|_{L^\infty}$ $\|v(t)\|_{W^{1,\infty}}$

χ μ 2-D.

1.1 - - $\|u(t)\|_{L^2}$

$\|(u+1)$ $(u+1)\|_{L^1}$ 24,38 ; , - - $\|u(t)\|^2_{L^2}$:

$$\begin{aligned}\|u(t)\|^2_{L^2}\le E(\chi,\mu)\Big\{&\|u_0\|^2_{L^2}+\frac{8\ \{1,\frac{2}{\chi}\}}{C_{GN}^2}+\frac{(r+1)}{\mu}\|u_0\|_{L^1}\\&+\frac{3\chi C_{GN}^2}{4}\left[\|u_0\|_{L^1}+\frac{(r+1)^2}{4\mu}|\Omega|\right]^4+\frac{(r+1)^3}{4\mu^2}|\Omega|+\frac{8r^3}{9\mu^2}|\Omega|\Big\},\end{aligned} \tag{1.7}$$

E fi (1.5) C_{GN} fi (1.6). A $\|u\|_{L^1}$, $\|\nabla v\|_{L^2}$

u^2 $|\Delta v|^2$, . L 2.3 3.1, 2-D G N

$y(t)=\|u(t)\|^2_{L^2}+a$:

$$y'(t)\le ky(t)z(t)+b,\quad z(t)=\|\Delta v(t)\|^2_{L^2}$$

$a,k,b>0$ ODI ,

(1.7).

Lemma 2.3. *For any $t \in [0, T\ \ )$, the nonnegative solution (u, v) to (1.1) satisfies*

$$\|u\|_{L^1} \le \|u_0\|_{L^1} + \frac{(r+1)^2}{4\mu}|\Omega| =: k_1 \tag{2.1}$$

and

$$\|\nabla v\|_{L^2}^2 \le \frac{2}{\mu}\Big[\|u_0\|_{L^1} + \frac{\mu}{2}\|\nabla v_0\|_{L^2}^2 + \frac{(r+2)^2}{4\mu}|\Omega|\Big] =: k_2. \tag{2.2}$$

Proof. u, v . , u-

N ,

$$\frac{}{t}\int_\Omega u = r\int_\Omega u - \mu\int_\Omega u^2 \le -\int_\Omega u + \frac{(r+1)^2}{4\mu}|\Omega|, \tag{2.3}$$

L^1- u (2.1).

, v- (1.1) $-\Delta v$ ' ,

$$\frac{1}{2}\frac{}{t}\int_\Omega |\nabla v|^2 + \frac{1}{2}\int_\Omega |\Delta v|^2 \le -\int_\Omega |\nabla v|^2 + \frac{1}{2}\int_\Omega u^2, \tag{2.4}$$

(2.3)

$$\frac{}{t}\int_\Omega (u + \frac{\mu}{2}|\nabla v|^2) + 2\int_\Omega (u + \frac{\mu}{2}|\nabla v|^2) \le \frac{(r+2)^2}{2\mu}|\Omega|.$$

G

$$\|u\|_{L^1} + \frac{\mu}{2}\|\nabla v\|_{L^2}^2 \le \|u_0\|_{L^1} + \frac{\mu}{2}\|\nabla v_0\|_{L^2}^2 + \frac{(r+2)^2}{4\mu}|\Omega|,$$

(2.2). □

3. Chemotaxis vs. logistic on boundedness in 2-D

I 2-D, ffi -

(1.1) - - 24,38 . I ,

27, L 3.4 ,

(1.1) χ μ, 1.1.

Lemma 3.1. *Given $\tau \in (0, T\ \ )$, then, for any $t \in [0, T\ \ - \tau)$, the solution (u, v) of the KS model (1.1) fulfills*

$$\int_t^{t+\tau}\int_\Omega u^2 \le \frac{(r+1)k_1}{\mu}\ \ \{\tau, 1\} =: k_3\ \ \{\tau, 1\}, \tag{3.1}$$

$$\int_t^{t+\tau}\int_\Omega |\nabla v|^2 \le k_2\ \ \{\tau, 1\} \tag{3.2}$$

and

$$\int_t^{t+\tau}\int_\Omega |\Delta v|^2 \le (k_3 + k_2)\ \ \{\tau, 1\} =: k_4\ \ \{\tau, 1\}. \tag{3.3}$$

Proof. F $t \in [0, T\ \ - \tau)$, u- (1.1) $\Omega \times (t, t+\tau)$ L 2.3,

$$\mu\int_t^{t+\tau}\int_\Omega u^2 \le r\int_t^{t+\tau}\int_\Omega u + \int_\Omega u \le (r+1)k_1\ \ \{\tau, 1\},$$

(3.1).

. (3.2) . (2.2). N , (2.4) $(t, t+\tau)$ (2.2) (3.1)

$$\int_t^{t+\tau}\int_\Omega |\Delta v|^2(s) \le \int_t^{t+\tau}\int_\Omega u^2(s) + \int_\Omega |\nabla v|^2(t) \le (k_3 + k_2) \quad \{\tau,$$

$$\begin{aligned}&\Big(\int_\Omega u^4\Big)^{\frac12}\Big(\int_\Omega |\Delta v|^2\Big)^{\frac12}\\&\le C_{GN}\|\nabla u\|_{L^2}\|u\|_{L^2}\|\Delta v\|_{L^2}+k_1^2C_{GN}\|\Delta v\|_{L^2}\\&\le \epsilon\|\nabla u\|_{L^2}^2+\frac{C_{GN}^2}{4\epsilon}\|u\|_{L^2}^2\|\Delta v\|_{L^2}^2+\|\Delta v\|_{L^2}^2+\frac{k_1^4C_{GN}^2}{4}.\end{aligned}\tag{3.7}$$

I (3.7) (3.6),

$$\begin{aligned}&\frac12\frac{\ }{t}\int_\Omega u^2+\int_\Omega|\nabla u|^2\\&\le\frac{\chi}{2}\Big[\epsilon\|\nabla u\|_{L^2}^2+\frac{C_{GN}^2}{4\epsilon}\|u\|_{L^2}^2\|\Delta v\|_{L^2}^2+\|\Delta v\|_{L^2}^2+\frac{k_1^4C_{GN}^2}{4}\Big]\\&\quad+\int_\Omega u^2(r-\mu u),\end{aligned}$$

,

$$\epsilon := \quad \{1,\frac{2}{\chi}\},\tag{3.8}$$

$$\begin{aligned}\frac{\ }{t}\int_\Omega u^2&\le\frac{\chi C_{GN}^2}{4\epsilon}\Big(\|u\|_{L^2}^2+\frac{4\epsilon}{C_{GN}^2}\Big)\|\Delta v\|_{L^2}^2+\frac{\chi k_1^4C_{GN}^2}{4}+\frac{8r^3}{27\mu^2}|\Omega|\\&=:k_5y(t)z(t)+k_6,\end{aligned}\tag{3.9}$$

$k_5=\frac{\chi C_{GN}^2}{4\epsilon}$, $k_6=\frac{\chi k_1^4C_{GN}^2}{4}+\frac{8r^3}{27\mu^2}|\Omega|$

$$y(t)=\|u\|_{L^2}^2+\frac{4\epsilon}{C_{GN}^2},\quad z(t)=\|\Delta v\|_{L^2}^2.\tag{3.10}$$

F $s\ge 0$ $t\ge s$, $(-k_5\int_s^t z(\lambda)\ \lambda)$ (3.9),

$$y(t)\le y(s)e^{k_5\int_s^t z(\sigma)\ \sigma}+k_6\int_s^t e^{k_5\int_\xi^t z(\sigma)\ \sigma}\ \xi,\quad \forall t\in[s,\infty)\cap[0,T\ ).\tag{3.11}$$

I (3.1) (3.3) L 3.1 , fi y z (3.10)

$$y(s_i)=\frac1\tau\int_{i\tau}^{(i+1)\tau}y(s)\ s\le(k_3+\frac{4\epsilon}{C_{GN}^2})\quad\{1,\frac1\tau\}=:k_7\quad\{1,\frac1\tau\}\tag{3.12}$$

$$\int_{i\tau}^{(i+1)\tau}z(s)\ s\le k_4\quad\{\tau,1\}\tag{3.13}$$

$s_i\in[i\tau,(i+1)\tau]$ $i<\frac{T}{\tau}-1$.
F , $t\in[0,\tau]$, $s=0$ (3.11) $i=0$ (3.13)

$$\begin{aligned}y(t)&\le y(0)\ ^{k_5\int_0^\tau z(\sigma)\ \sigma}+k_6\int_0^\tau\ ^{k_5\int_0^\tau z(\sigma)\ \sigma}\ \xi\\&\le(y(0)+k_6)\quad\{\tau,1\}\ ^{k_5k_4\quad\{\tau,1\}}.\end{aligned}\tag{3.14}$$

N , $t\in[\tau,2\tau]$, $t<T\ $, $s=s_0\in[0,\tau]$ (3.11) (3.12) (3.13)

$$
\begin{aligned}
y(t) &\le y(s_0) \;\; {}^{k_5 \int_{s_0}^{t} z(\sigma) \;\; \sigma} + k_6 \int_{s_0}^{t} \;\; {}^{k_5 \int_{s_0}^{t} z(\sigma) \;\; \sigma} \;\; \xi \\
&\le y(s_0) \;\; {}^{k_5 \int_0^{2\tau} z(\sigma) \;\; \sigma} + k_6 \int_0^{2\tau} \;\; {}^{k_5 \int_0^{2\tau} z(\sigma)}
\end{aligned}
$$

Proof. I , - - v (1.1)

$$v(t) = {}^{t(\Delta-1)}v_0 + \int_0^t {}^{(t-s)(\Delta-1)}u(s) \; s. \tag{3.19}$$

N , - L^p L^q N , . 2,31 , , $1 \le q \le p \le \infty$, fi $k_8, k_9, k_{10} > 0$

$$\| {}^{t\Delta} w\|_{L^p} \le k_8 \left(1 + t^{-\frac{n}{2}(\frac{1}{q}-\frac{1}{p})}\right) \|w\|_{L^q}, \quad \forall t > 0 \tag{3.20}$$

$$\|\nabla {}^{t\Delta} w\|_{L^q} \le k_9 \|\nabla w\|_{L^\infty}, \quad \forall t > 0 \tag{3.21}$$

$$\|\nabla {}^{t\Delta} w\|_{L^p} \le k_{10} \left(1 + t^{-\frac{1}{2}-\frac{n}{2}(\frac{1}{q}-\frac{1}{p})}\right) {}^{-\lambda_1 t} \|w\|_{L^q}, \quad \forall t > 0. \tag{3.22}$$

H , $\lambda_1 (> 0)$ fi $-\Delta$. (3.20), (3.21) (3.22) (3.19) p, q (3.17), (3.18). □

Lemma 3.5. *The u-component of the unique global-in-time classical solution to the KS minimal chemotaxis-logistic model* (1.1) *satisfies the uniform estimate*

$$\|u(t)\|_{L^3} \le C\left[1 + \frac{1}{\mu} + \frac{\chi^{\frac{8}{3}}}{\mu} M^{\frac{8}{3}}(\chi,\mu) E^{\frac{8}{3}}(\chi,\mu)\right], \tag{3.23}$$

for all $t \in (0, \infty)$ and for some C depending on u_0, v_0, r and $|\Omega|$, where M and E are defined by (1.4) *and* (1.5)*, respectively.*

Proof. B L^2- u (3.5) $\tau = 1$, L 3.4 $n = 2$ $p = 2$, $1 < q < \infty$,

$$\|\nabla v(t)\|_{L^q} \le C M(\chi,\mu) E(\chi,\mu). \tag{3.24}$$

M u- (1.1) u^2, 8 ' ,

$$\begin{aligned}
&\frac{1}{3}\frac{}{t}\int_\Omega u^3 + 2\int_\Omega u|\nabla u|^2 \\
&= 2\chi \int_\Omega u^2 \nabla u \nabla v + \int_\Omega (ru^3 - \mu u^4) \\
&\le 2\int_\Omega u|\nabla u|^2 + \frac{\chi^2}{2}\int_\Omega u^3|\nabla v|^2 + \int_\Omega (ru^3 - \mu u^4) \\
&\le 2\int_\Omega u|\nabla u|^2 + \frac{\mu}{2}\int_\Omega u^4 + \frac{3^3\chi^8}{2\cdot 4^4\mu^3}\int_\Omega |\nabla v|^8 + \int_\Omega (ru^3 - \mu u^4),
\end{aligned}$$

$ru^3 - \frac{\mu}{2}u^4 \le -\frac{1}{3}u^3 + \frac{3^3(r+\frac{1}{3})^4}{2^5\mu^3}$

$$\frac{}{t}\int_\Omega u^3 + \int_\Omega u^3 \le \frac{3^4\chi^8}{2\cdot 4^4\mu^3}\|\nabla v\|_{L^8}^8 + \frac{3^4(r+\frac{1}{3})^4}{2^5\mu^3}|\Omega|.$$

G ,

$$\|u\|_{L^3}^3 \le \|u_0\|_{L^3}^3 + \frac{3^4\chi^8}{2\cdot 4^4\mu^3} {}_{t\in(0,\infty)} \|\nabla v(t)\|_{L^8}^8 + \frac{3^4(r+\frac{1}{3})^4}{2^5\mu^3}|\Omega|,$$

, (3.24) $q = 8$, (3.23). □

Proof of Theorem 1.1

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