Integrable pair-transition-coupled nonlinear Schrödinger equations

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I. INTRODUCTION

[1] . [2] . [3] . [15] . [16] . [7] , [1] [1] .

[3] , -E .

II. THE COUPLED NONLINEAR SCHRÖDINGER EQUATIONS WITH PARTICLE TRANSITION

-E $H = \sum_j \frac{\hbar^2}{2m}\partial_x^2 q_j q_j^\dagger + \frac{g_{j,j}}{2} n_j n_j + g_{j,3-j} n_j n_{3-j} + J_1(q_j^\dagger q_{3-j} + q_{3-j}^\dagger q_j) + \frac{J_2}{2}(q_j^\dagger q_j^\dagger q_{3-j} q_{3-j} + q_{3-j}^\dagger q_{3-j}^\dagger q_j q_j)$ $n_j = q_j^\dagger q_j$ $\dagger$ $g_{i,i}$ $g_{3-i,i}$ $(i = 1,2)$. J_1 J_2 [20,21] . , $J_{1,2}$ [5] . -E [22,23] . $J_1 = 0$ $J_2 \neq 0$. $g_{j,3-j} = 2g_{j,j} = 2J_2$. $g_{j,j} = -\sigma$ $(\sigma = \pm 1$)

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′ n n n ′ n
$\hbar(\partial q_j/\partial t) = q_j, H$. n
n n $\langle q_j \rangle = q_j$, n n
n - n n $m = \hbar = 1$

$$\begin{aligned} q_{1,t} + \tfrac{1}{2} q_{1,xx} + \sigma(|q_1|^2 + 2|q_2|^2) q_1 + \sigma q_2^2 q_1 = 0, \\ q_{2,t} + \tfrac{1}{2} q_{2,xx} + \sigma(2|q_1|^2 + |q_2|^2) q_2 + \sigma q_1^2 q_2 = 0, \end{aligned} \tag{1}$$

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′ n ′ -
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n ′

n n n n $q_1, q_2,$ n $q_1\ 1\ , q_2\ 1$

$$q_1\ 1\ = q_1 - \frac{\delta\psi_1\phi_1}{(|\phi_1|^2+\sigma|\psi_1|^2)/(\lambda_1-\overline{\lambda_1})},$$
$$q_2\ 1\ = q_2 - \frac{\psi_1\phi_1}{(|\phi_1|^2+\sigma|\psi_1|^2)/(\lambda_1-\overline{\lambda_1})}. \qquad (7)$$

n $\Omega_1 \neq \delta\Sigma\Omega_1,\ \Phi_1 = A(x,t)\ \Omega_1,\delta\Sigma\Omega_1\ ,$ n n-
' n '

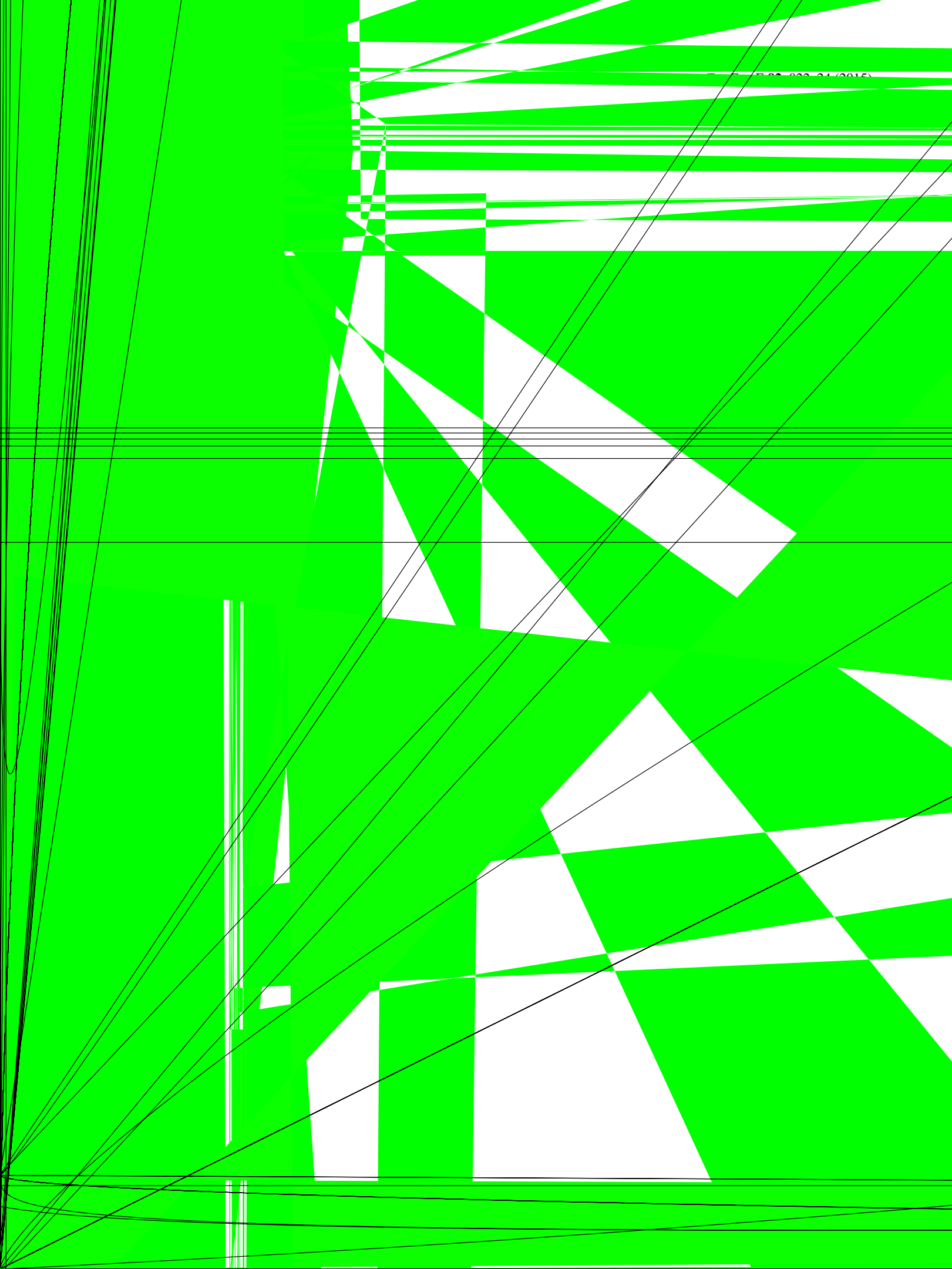
$$T_2 = I - \frac{\lambda_1-\overline{\lambda_1}}{\lambda-\overline{\lambda_1}}\Phi_1(\Phi_1^\dagger J\Phi_1)^{-1}\Phi_1^\dagger J,$$
$$\Phi_1 = \begin{bmatrix}\phi_1 & \varphi_1 & \chi_1 & \psi_1\\ \delta\varphi_1 & \delta\phi_1 & \delta\psi_1 & \delta\chi_1\end{bmatrix}^T, \qquad (\ )$$

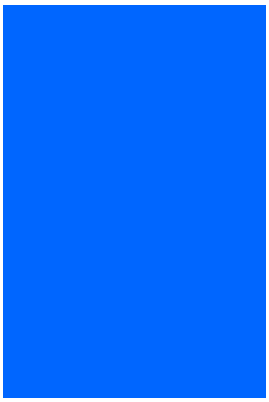
n n n n q_1, q_2 n $q_1\ 1\ , q_2\ 1$

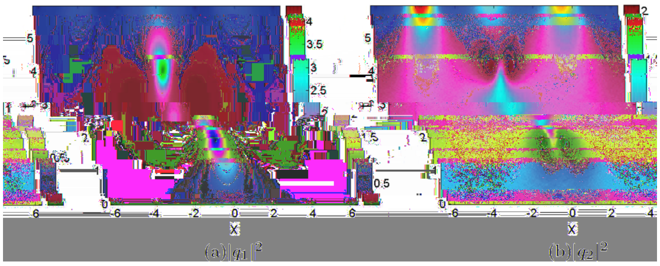
$$q_1\ 1\ = q_1 - N_1 - N_2, \quad q_2\ 1\ = q_2 + N_1 - N_2, \qquad (\ )$$

$$N_1 = \frac{(\varphi_1-\phi_1)(\psi_1-\chi_1)}{(|\varphi_1-\phi_1|^2+\sigma|\psi_1-\chi_1|^2)/(\lambda_1-\overline{\lambda_1})},$$
$$N_2 = \frac{(\varphi_1+\phi_1)(\psi_1+\chi_1)}{(|\varphi_1+\phi_1|^2+\sigma|\psi_1+\chi_1|^2)/(\lambda_1-\overline{\lambda_1})}.$$

' 7) n (







. 5. (n n) n n n n n n n n n $t = -3$ n . 4 n $(1 + 0.01 Random$ $-1,1$). n n n .

n n , n n . 5, n n n $t = -3$ n . 4 n n n $(1 + 0.01 Random$ $-1,1$) . n n n n n. n , n n 43 . n n n n n n n n n 44 46 .

VI. POSSIBILITIES TO OBSERVE THESE NONLINEAR EXCITATIONS

n , n n n n n n n n n n 4 ,4 . n n n - , - n , - n , n n - n n n n n n -E n n n n n n n n n 4 . n n n n n n n n n n n n n n n ( ) n n n n - n n -E n n n n 20,21 - - n n n n n n . 2 ,2 . n n n n - n n n n n n n n n n . , n n n n n n n n n n n n . , n n - n n n n n n n n n n n n . n , . n n n . 4 n - n n n , q_1 n q_2.

, n n n n n n n q_2. q_1 q_2 n n n $q_2 \to q_1$ n n. n n n n n

22,23 . n 7. n n n $N = 5 \times 10^4$. $a_{i,j} (i, j = 1,2)$ s- - n n n n n n . n $a_{1,2} = a_{2,1} = 1.6$ n n $a_{2,2} = a_{1,1} =$ 0. n , n n- n, s- n n n n n n n $U_{j,j} = 4\pi\hbar^2 a_{j,j}/m$ (m), n n n n n n n n n n $U_{j,3-j} = 4\pi\hbar^2 a_{j,3-j}/m$. n n n n n n n $N U_{1,1}$, n n n n 2.0 μ n 0.5 , , n n n n n E . (1). n (15) n $\theta =$ (3/5), $x_1 = 0$, n $t_1 = 1$ n n n n n n n n n n n n n n n n n n

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