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High-order rogue waves in vector nonlinear Schrödinger equations

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We study the dynamics of high-order rogue waves (RWs) in two-component coupled nonlinear Schrödinger equations. We find that four fundamental rogue waves can emerge from second-order vector RWs in the coupled system, in contrast to the high-order ones in single-component systems. The distribution shape can be quadrilateral, triangle, and line structures by varying the proper initial excitations given by the exact analytical solutions. The distribution pattern for vector RWs is more abundant than that for scalar rogue waves. Possibilities to observe these new patterns for rogue waves are discussed for a nonlinear fiber.

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Introduction. Rogue wave (RW) is the name given by oceanographers to isolated large-amplitude waves which occur more frequently than expected for normal, Gaussian-distributed, statistical events [1–3]. It depicts a unique event that seems to appear from nowhere and disappear without a trace, and can appear in a variety of different contexts [4–7]. RWs have been observed experimentally in nonlinear optics [8,9], water wave tanks [10], and even in plasma systems [11]. These experimental studies suggest that the rational solutions of related dynamics equations can be used to describe these RW phenomena [12,13]. Moreover, there are many different pattern structures for high-order RWs [14–16], which can be understood as a nonlinear superposition of the fundamental RW (the first-order RW). Recently, many efforts were devoted to classifying the hierarchy for each order RW [17,18], since the superpositions are nontrivial and admit only a fixed number of elementary RWs in each high-order solution.

Recent studies were extended to RWs in multicomponent coupled systems, since complex systems usually involve more than one component [19–21]. For the coupled system, the usual coupled effects are cross-phase modulation. The linear stability analysis of the coupled system indicates that the cross-phase modulation term can vary the instability regime characters [22–24]. Moreover, for scalar systems, the velocity of the background field has no real effect on the pattern structure for RWs, since the corresponding solutions can be correlated through Galileo transformation. But for a coupled system, the relative velocity between different component fields has real physical effects, and cannot be erased by any trivial transformation. Therefore, the extended studies on vector RWs are nontrivial and meaningful. Recently, some novel patterns for RWs were presented in the coupled systems, such as dark RWs [25,26], the interaction between RWs and other nonlinear waves [26–28], a four-petaled flower structure [29,30], and so on. These studies indicate that there are very abundant pattern dynamics for RWs in the multicomponent coupled systems, which are quite distinctive from the ones in scalar systems.

Very recently, high-order RWs [31,32] were excited successfully in a water wave tank. This suggests that a high-order analytic RW solution is meaningful physically and can be realized experimentally [33]. However, as far as we know, the high-order vector RW has not been taken seriously until now. In [34], the authors consider the high-order RW solutions, which can be reduced into scalar ones, by a modifi1(into1.15e)-001 Tc4gh-

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The standard DT [36] for linear system (2) is

$$\Phi[1] = T\Phi, \quad T = I + \frac{\bar{\lambda}_1 - \lambda_1}{\lambda - \bar{\lambda}_1}\frac{\Phi_1\Phi_1^\dagger}{\Phi_1^\dagger\Phi_1},$$
$$Q[1] = Q + i(\bar{\lambda}_1 - \lambda_1)[P_1,\Lambda], \tag{3}$$

where Φ_1 is a special solution for system (1

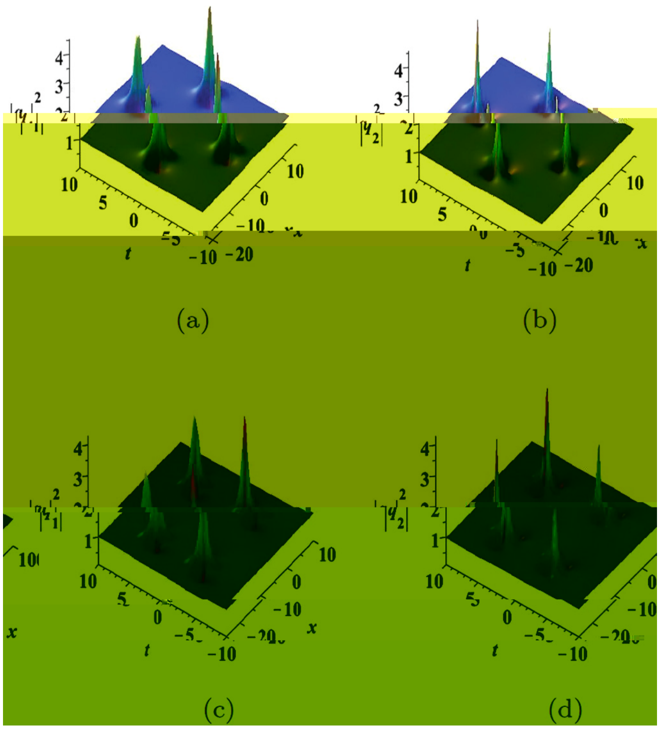

FIG. 1. (Color online) (a), (b) The rhombus structure for the second-order vector RW which contains four fundamental ones. The parameters are $f_1 = 0$, $f_2 = 0$, $g_1 = 1$, $g_2 = 0$, $h_1 = 0$, $h_2 = 10\,000$. (c), (d) The rectangle structure for the second-order vector RW which contains four fundamental ones. The parameters are $f_1 = 0$, $f_2 = 0$, $g_1 = 1$, $g_2 = 1000$, $h_1 = 10$, $h_2 = 0$.

As an example, we exhibit the dynamics behavior of a second-order vector RW solution. Since the expressions of the high-order RW solution are quite complicated, we will present them elsewhere.

The dynamics of second-order vector rogue waves. There are six free parameters in the generalized second-order RW solution, denoted by f_j, g_j, and h_j ($j = 1,2$), which can be used to obtain different types or patterns for the rogue-wave dynamics. We find that there are mainly two kinds of RW solutions which correspond to four fundamental RWs and six fundamental ones obtained by setting $f_1 = 0$ and f_1

FIG. 3. (Color online) (a), (b) The pentagon structure for the second-order vector RW which contains six fundamental ones. The parameters are $f_1 = 1, f_2 = 0, g_1 = 0, g_2 = 0, h_1 = 0, h_2 = 10\,000$. (c), (d) The rectangle structure for the second-order vector RW which contains six fundamental ones. The parameters are $f_1 = 1, f_2 = 0$, $g_1 = 0, g_2 = 0, h_1 = 100, h_2 = 0$.

FIG. 4. (Color online) (a), (b) The triangle structure for the second-order vector RW which contains six fundamental ones. The parameters are $f_1 = 1, f_2 = 0, g_1 = 10, g_2 = 0, h_1 = 0, h_2 = 0$. (c), (d) The line structure for the second-order vector RW which contains six fundamental ones. The parameters are $f_1 = 1$, $f_2 = 0$, $g_1 = 0$, $g_2 = 0, h_1 = 0, h_2 = 0$.

distribution plane can be varied too. This case is similar to the pentagon structure of the third-order RW of a scalar NLS equation [17,18]. The six RWs can be arranged with the rectangle structure by varying the parameters, such as the one in Figs. 3(c) and 3(d). The structure is similar to the one in Figs. 1(c) and 1(d). But there is a new RW inside the rectangle, which is formed by the interaction of the other two fundamental RWs.

The six RWs can be also arranged with the triangle structure, shown in Figs. 4(a) and 4(b). In this case, there are two fundamental RWs and a new RW to form a triangle. The new RW is formed by the interaction between the other four fundamental RWs. Moreover, the RWs can be arranged with a line structure too, shown in Figs. 4(c) and 4(d). There should be six fundamental RWs arranged in one line case, but it is very complicated to derive the case since the parameters are too many to be managed well. We just show a particular one of the line cases.

Possibilities to observe these vector rogue waves. It is expected that these vector RWs could be observed in two-mode nonlinear fibers [8,9]. As an example, we consider the case that the operation wavelength of each mode is nearly 1.55 μm, the group-velocity dispersion (GVD) coefficients are -20 ps^2 km^{-1} in the anomalous regime, and the Kerr coefficients are nearly 1.1 W^{-1} km^{-1}, corresponding to the self-focusing effect in the fiber [37]. The unit in x direction will be denoted as 0.23 ps, and the one in t will be denoted as 0.55 km. One can introduce two distinct modes to the nonlinear fiber operating in the anomalous GVD regime [38,39]. The spontaneous development of RWs seeded from some perturbation should be on the continuous waves as those in [8,9]. The continuous-wave background intensities in the two modes should be equal (assume to be 1 W), and the frequency difference between the two modes should be 0.23 ps^{-1} to observe the vector RW. We can manipulate the initial perturbation approaching the ideal initial condition to observe these vector RW patterns. The ideal initial condition including intensity and phase profiles can be given by the exact second-order vector RW solution with given parameters in the scaled units. Moreover, the initial intensity and phase profiles can be made through the related modulation operator [9,37]. The vector RWs could be observed in the nonlinear fiber while approaching the ideal initial excitation form presented here [9].

Conclusions. We present a generalized RW solution of the CNLSE which can be used to obtain an arbitrary-order vector RW. We find that there are mainly two kinds of rogue-wave solutions for the second-order vector RW in CNLSE, which correspond to four fundamental RWs and six fundamental ones. Based on these results, we expect that there are many more abundant exotic patterns for RWs in NLSEs with three or more components.

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