1

1.1

$\mathbb{R}^3$ n

$\mathbb{R}^3$ $\{p; e_1(p), e_2(p), e_3(p)\}$

$$\begin{cases} \dfrac{\partial e_1}{\partial v} = b_1^1 e_1 + \ b_1^2 e_2 + \ b_1^3 e_3 \\ \dfrac{\partial e_2}{\partial v} = b_2^1 e_1 + \ b_2^2 e_2 + \ b_2^3 e_3 \\ \dfrac{\partial e_3}{\partial v} = b_3^1 e_1 + \ b_3^2 e_2 + \ b_3^3 e_3 \end{cases}$$

$\frac{\partial}{\partial}$

$$\frac{\partial}{\partial v}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \ B \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$$

$$B = \begin{pmatrix} b_1^1 & b_1^2 & b_1^3 \\ b_2^1 & b_2^2 & b_2^3 \\ b_3^1 & b_3^2 & b_3^3 \end{pmatrix}$$

e_1, e_2, e_3 (a_{11}, a_{12}, a_{13}) (a_{21}, a_{22}, a_{23}) (a_{31}, a_{32}, a_{33})

$$\frac{\partial}{\partial v} A = \ BA$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

v

$$A = BA$$

$B = (\omega\ \ )$ ω $\omega\ \ (v) = \ b$ B

ω

$$\{p; e_1(p), e_2(p), e_3(p)\}$$

ω_i^j

$$B = B(\frac{\partial}{\partial x}) = (\omega \ (\frac{\partial}{\partial x})) = (\tfrac{\partial}{\partial x^i} b \ )$$

$\frac{\partial}{\partial x^i} b$ b B (j,k) b

$$\frac{\partial}{\partial x} A = BA, \quad i = 1,2,3$$

$$\frac{\partial^2}{\partial x \ \partial x} A = \frac{\partial^2}{\partial x \ \partial x} A$$

$$\frac{\partial B}{\partial x} A + B BA = \frac{\partial B}{\partial x} A + B BA$$

A

$$\frac{\partial B}{\partial x} + B B = \frac{\partial B}{\partial x} + B B$$

$$\omega = \sum_{=1}^{3} b \quad x$$

$$\omega = b \quad x$$

B

$$B = B \ x$$

$$\circ \quad \equiv 0$$

$$0 = \quad A = (\ B)A - B \wedge \ A$$

$\wedge$ $\otimes$ A

$$(\ B)A - (B \wedge B)A = 0$$

A

$$B = B \wedge B$$

$$(\ B \ x \ ) = (B \ x \ ) \wedge (\ B \ x \ )$$

$$\frac{\partial B}{\partial x} \ x \wedge \ x \ = B \ B \ x \wedge \ x$$

$$\frac{\partial\, B}{\partial x}\;\; x \;\wedge\;\; x \;\;=\; B\, B\;\; x \;\wedge\;\; x$$

$$\sum_{<}\Big(\frac{\partial\, B}{\partial x} - \frac{\partial\, B}{\partial x}\Big)\;\; x \;\wedge\;\; x \;\;= \sum_{<}\big(\; B\, B - \;\; B\, B\big)\;\; x \;\wedge\;\; x$$

$x \;\wedge\;\; x$

$$\frac{\partial\, B}{\partial x} - \frac{\partial\, B}{\partial x} = \;\; B\, B - \;\; B\, B \quad \forall i, j$$

$$\circ \quad \equiv 0$$

$$\begin{cases} \;\; A = & B A \\ \;\; B = & B \wedge B \end{cases}$$

$$\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = B \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$$

$B = (\omega \;\;)$ e

$$e \;\; = \omega \;\; \otimes e$$

B $(\omega \;\;)$

$$\omega \;\; = \omega \;\; \wedge \omega$$

$$\begin{cases} \;\; e \;\; = & \omega \;\; \otimes e \\ \;\; \omega \;\; = & \omega \;\; \wedge \omega \end{cases}$$

g ∇

$\mathbb{R}$

$\mathbb{R}^3$ S

1.2

M n g

$T\ M$ M p $T\ M$ g $\{e_1, e_2, \cdots, e\ \}$

p M

$\{e_1, e_2, \cdots, e\ \}$ $v \in T\ M$

n

$$\nabla\ e\ =\ a\ e$$

∇ g a v

ω

$v \in T\ M$ ω v $\omega\ (v) =\ a$

$$\nabla e\ = \omega\ \otimes e$$

g

$$\begin{aligned}\nabla\ g(e\ , e\ ) =& g(\nabla\ e\ , e\ ) + g(e\ , \nabla\ e\ )\\ =& g(\omega\ (v) e\ , e\ ) + g(e\ , \omega\ (v) e\ )\\ =& \omega\ (v) + \omega\ (v)\end{aligned}$$

$g(e\ , e\ ) \equiv \delta$ $\nabla\ g(e\ , e\ ) \equiv 0$ $\omega\ = -\omega$

$$\nabla\ _i e\ - \nabla\ _j e\ - [e\ , e\ ] = 0$$

$[e\ , e\ ]$ $e\ e\ - e\ e$ $[e\ , e\ ] = c\ \ e$

$$\omega\ (e\ ) - \omega\ (e\ ) = c$$

$\{e\ \}$

$$\nabla e\ = \omega\ \otimes e$$

ω

$\omega\ = -\omega$

$\omega\ (e\ ) - \omega\ (e\ ) = c$

e ω

1.3

θ

T^*M e $\theta\ (e\ ) = \delta$

∇ $\tilde{\omega}$ $\nabla\theta\ = \tilde{\omega}\ \otimes \theta$

$$\nabla(\theta\ (e\ )) = (\nabla\theta\ )(e\ ) + \theta\ (\nabla e\ ) = \tilde{\omega}\ \theta\ (e\ ) + \omega\ \theta\ (e\ )$$

$$\tilde{\omega}\ \delta\ + \omega\ \delta\ = \tilde{\omega}\ + \omega\ = 0$$

$\{\theta\ \}$

$$\nabla\theta\ = -\omega\ \otimes \theta$$

θ

$$\omega\ = \Gamma\ \ \theta$$

Γ

$$\Gamma\ \ \theta\ (e\ ) - \Gamma\ \ \theta\ (e\ ) = c$$

$$\Gamma\ \ - \Gamma\ \ = c$$

$$\nabla\theta\ = -\omega\ \otimes \theta$$

$\omega\ = -\omega$

$\Gamma\ \ - \Gamma\ \ = c$

1.4

∇

1. α 1 X, Y

$$\alpha(X, Y) = X\alpha(Y) - Y\alpha(X) - \alpha([X, Y])$$

α f

2. φ k $Y_0, Y_1, \cdots, Y$

$$(\ \varphi)(Y_0, Y_1, \cdots, Y \ )$$

$$= \sum_{=0} (-1) \ Y \ (\varphi(Y_0, \cdots, \hat{Y}, \cdots, Y \ )) + \sum_{0 \le \ < \ \le} (-1)^{\ +} \ \varphi([Y \ , Y \ ], Y_0, \cdots, \hat{Y}, \cdots, \hat{Y}, \cdots, Y \ )$$

∇

1. ∇ $= \theta \ \wedge \nabla_{\ i}$ $\{e \ \}$ $\{\theta \ \}$

$\theta \ \wedge \nabla_{\ i}$

α α $\theta \ \wedge \nabla_{\ i} \alpha$ ∇

$$\begin{aligned} \alpha(e \ , e \ ) =& e \ \alpha(e \ ) - e \ \alpha(e \ ) - \alpha([e \ , e \ ]) \\ =& (\nabla_{\ k} \alpha)(e \ ) + \alpha(\nabla_{\ k} e \ ) - (\nabla_{\ l} \alpha)(e \ ) - \alpha(\nabla_{\ l} e \ ) - \alpha([e \ , e \ ]) \\ =& (\nabla_{\ k} \alpha)(e \ ) - (\nabla_{\ l} \alpha)(e \ ) + \alpha(\nabla_{\ k} e \ - \nabla_{\ l} e \ - [e \ , e \ ]) \\ =& (\nabla_{\ k} \alpha)(e \ ) - (\nabla_{\ l} \alpha)(e \ ) \end{aligned}$$

$$(\theta \ \wedge \nabla_{\ i} \alpha)(e \ , e \ ) = \theta \ (e \ )(\nabla_{\ i} \alpha)(e \ ) - \theta \ (e \ )(\nabla_{\ i} \alpha)(e \ ) = (\nabla_{\ k} \alpha)(e \ ) - (\nabla_{\ l} \alpha)(e \ )$$

$= \theta \ \wedge \nabla_{\ i}$

$\theta \ \wedge \nabla_{\ i}$ k $\square$

$\theta \ \wedge \nabla_{\ i}$

f $\theta \ \wedge \nabla_{\ i} f =$ f

$(\theta \ \wedge \nabla_{\ i}) \circ (\theta \ \wedge \nabla_{\ i}) = 0$

$\theta \ \wedge \nabla_{\ i}$

$$\nabla \theta \ = -\omega \ \otimes \theta$$

$$\theta \ = \theta \ \wedge \nabla_{\ i} \theta \ = \theta \ \wedge (-\omega \ (e \ )) \theta \ = -\omega \ (e \ ) \theta \ \wedge \theta \ = -\omega \ \wedge \theta$$

ω

$$\theta \ = -\omega \ \wedge \theta$$

ω

$\omega \;\; = \Gamma \;\; \theta$

$$\theta \;\; = -\Gamma \;\; \theta \;\; \wedge \theta \;\; = -\sum_{<} (\Gamma \quad - \Gamma \quad) \theta \;\; \wedge \theta$$

$\theta \;\; \wedge \theta \;, k < i$ $n \times \frac{(\; -1)}{2}$ Γ

ω i, j

$$\Gamma \;\; \theta \;\; = -\Gamma \;\; \theta$$

$\Gamma \quad = -\Gamma$ $\Gamma \quad \equiv 0$ Γ $n \times \frac{(\; -1)}{2}$

Γ ω

1.5

$\{e \;\}$ $\{\theta \;\}$

$\tilde{\omega}$ $\{\tilde{e} \;\}$

$\{\tilde{\theta} \;\}$ $\tilde{\omega}$

X, Y

$$\nabla \quad \nabla \quad e \;\; - \nabla \quad \nabla \quad e \;\; - \nabla_{[\;\; , \;\;]} e \;\; = R(X, Y) e$$

$R(X, Y)$ $(1, 3)$

$$\nabla e \;\; = \omega \;\; \otimes e$$

$$\nabla \quad (\omega \;\; (Y) e \;\;) - \nabla \quad (\omega \;\; (X) e \;\;) - (\omega \;\; ([X, Y]) e \;\;) = R(X, Y) e$$

$$\big(\nabla \quad \omega \;\; (Y) - \nabla \quad \omega \;\; (X) - \omega \;\; ([X, Y])\big) \, e \;\; + \big(\omega \;\; (Y) \omega \;\; (X) - \omega \;\; (X) \omega \;\; (Y)\big) \, e \;\; = R(X, Y) e$$

$$\nabla \quad \omega \;\; (Y) - \nabla \quad \omega \;\; (X) - \omega \;\; ([X, Y]) = (\;\; \omega \;\;)(X, Y)$$

$$\omega \;\; (Y) \omega \;\; (X) - \omega \;\; (X) \omega \;\; (Y) = -\omega \;\; \wedge \omega \;\; (X, Y)$$

$$\big((\;\; \omega \;\;)(X, Y) - \omega \;\; \wedge \omega \;\; (X, Y)\big) \, e \;\; = R(X, Y) e$$

$R(X,Y)e = \Omega\ (X,Y)e$ Ω

$$\omega - \omega \wedge \omega = \Omega$$

$\tilde{\omega}$

$\tilde{\omega} - \tilde{\omega} \wedge \tilde{\omega} = \tilde{\Omega}$ $\{\tilde{e}\ \}$ $\tilde{\omega}$

$\{e\ \}$

ω ω

$\{\tilde{e}\ \}$ $\tilde{\omega}$ $\{e\ \}$

$$\tilde{e} = a\ e$$

$(a\ )$ $\{\tilde{e}\ \}$ $(a\ )$

$$\nabla\tilde{e} = \tilde{\omega} \otimes \tilde{e}$$

$$\nabla(a\ e\ ) = \tilde{\omega} \otimes (a\ e\ )$$

$$(\ a\ ) \otimes e + a\ \nabla e = a\ \tilde{\omega} \otimes e$$

$$(\ a\ ) \otimes e + a\ \omega \otimes e = a\ \tilde{\omega} \otimes e$$

$$(\ a\ ) \otimes e + a\ \omega \otimes e = a\ \tilde{\omega} \otimes e$$

$$a + a\ \omega = a\ \tilde{\omega}$$

$$a = a\ \tilde{\omega} - a\ \omega$$

a

$\circ = 0$

a

$$a = (a\ \tilde{\omega} - a\ \omega\ ) = (\ a\ ) \wedge \tilde{\omega} + a\ \tilde{\omega} - (\ a\ ) \wedge \omega - a\ \omega$$

$$\begin{aligned}
&(a\ \tilde{\omega} - a\ \omega\ ) \wedge \tilde{\omega} + a\ (\tilde{\omega} \wedge \tilde{\omega} + \tilde{\Omega}\ ) - (a\ \tilde{\omega} - a\ \omega\ ) \wedge \omega - a\ (\omega \wedge \omega + \Omega\ ) \\
&= a\ \tilde{\omega} \wedge \tilde{\omega} + a\ \tilde{\omega} \wedge \tilde{\omega} \\
&- (a\ \omega \wedge \tilde{\omega} + a\ \tilde{\omega} \wedge \omega\ ) \\
&+ a\ \omega \wedge \omega - a\ \omega \wedge \omega \\
&+ a\ \tilde{\Omega} - a\ \Omega
\end{aligned}$$

a a

$$a\ \tilde{\Omega}\ a = \Omega$$

$a \equiv 0$

$$\begin{cases} \theta = -\omega \wedge \theta \\ \omega = \omega \wedge \omega + \Omega \end{cases}$$

ω Ω i, j ω

1.6

$$\theta = 0$$

$$\begin{aligned} (-\omega \wedge \theta) &= -(\ \omega\ ) \wedge \theta + \omega \wedge \ \theta \\ &= -(\ \omega\ ) \wedge \theta - \omega \wedge \omega \wedge \theta \\ &= -(\ \omega + \omega \wedge \omega\ ) \wedge \theta \end{aligned}$$

$$(\ \omega - \omega \wedge \omega\ ) \wedge \theta = 0$$

$\omega - \omega \wedge \omega = 0$ $\omega - \omega \wedge \omega =$
Ω $R(X, Y)e = \Omega\ (X, Y)e$

$$\Omega = R\ \ \theta \wedge \theta$$

$R(e\ , e\ )e = R\ \ e$

$$\Omega \wedge \theta = R\ \ \theta \wedge \theta \wedge \theta$$

$$R(X, Y)Z + R(Y, Z)X + R(Z, X)Y = 0$$

$$R\ \ + R\ \ + R\ \ = 0$$

$$\Omega \wedge \theta = 0$$

$\circ\ \ = 0$ $\circ\ \ = 0$

2

2.1 Gauss-Codazzi

$\mathbb{R}^3$ S $\{\vec{r}, e_1, e_2, e_3\}$

e_1, e_2 S $e_3 = \vec{n}$

S

$\theta^1, \theta^2, \theta^3$ $\omega \ = -\omega$ $\mathbb{R}^3$

$$\begin{cases} \theta \ = - \omega \ \wedge \theta \\ \omega \ = \omega \ \wedge \omega \end{cases}$$

i, j $1, 2, 3$

S $\theta^1, \theta^2, \theta^3$ ω S (u, v)

$e_3 = \vec{n}$ S $\theta^3| \ \equiv 0$

$\omega_3^3 \equiv 0$ S

$$\begin{cases} \theta^\beta = - \omega_\alpha^\beta \wedge \theta^\alpha \\ \omega_\alpha^\beta = \omega_\alpha^\gamma \wedge \omega_\gamma^\beta + \omega_\alpha^3 \wedge \omega_3^\beta \end{cases}$$

$\alpha, \beta = 1, 2$ 1 2

$$\omega_\alpha^3 = \omega_\alpha^\gamma \wedge \omega_\gamma^3$$

$\mathbb{R}^3$

θ^1, θ^2 S

$$\omega_\alpha^\beta = \omega_\alpha^\gamma \wedge \omega_\gamma^\beta + \Omega_\alpha^\beta$$

$$\Omega_\alpha^\beta = \omega_\alpha^3 \wedge \omega_3^\beta$$

S

S S Ω_α^β

S

$$\begin{cases} \theta^\beta = - \omega_\alpha^\beta \wedge \theta^\alpha \\ \omega_\alpha^\beta = \omega_\alpha^\gamma \wedge \omega_\gamma^\beta + \Omega_\alpha^\beta \end{cases}$$

$$\begin{cases} \Omega_\alpha^\beta = \omega_\alpha^3 \wedge \omega_3^\beta \\ \omega_\alpha^3 = \omega_\alpha^\gamma \wedge \omega_\gamma^3 \end{cases}$$

S

Ω_α^β $\omega_\alpha^3 \wedge \omega_3^\beta$

α, β $\alpha = 1, \beta = 2$

$$\Omega_1^2 = R_{\gamma\eta 1}^2 \theta^\gamma \wedge \theta^\eta = 2R_{121}^2 \theta^1 \wedge \theta^2$$

R_{121}^2 $k < l$

$\gamma < \eta$ $\omega_\alpha^3 = h_{\alpha\gamma}\theta^\gamma$

$$\begin{cases} \omega_1^3 = h_{11}\theta^1 + h_{12}\theta^2 \\ \omega_2^3 = h_{21}\theta^1 + h_{22}\theta^2 \end{cases}$$

$$\omega_1^3 \wedge \omega_3^2 = -\omega_1^3 \wedge \omega_2^3 = -(h_{11}\theta^1 + h_{12}\theta^2) \wedge (h_{21}\theta^1 + h_{22}\theta^2) = -(h_{11}h_{22} - h_{12}h_{21})\theta^1 \wedge \theta^2$$

$$-2R_{121}^2 = h_{11}h_{22} - h_{12}h_{21}$$

$$R_{121}^2 = R_{1221}$$

$$\begin{cases} \omega_1^3 = \omega_1^2 \wedge \omega_2^3 \\ \omega_2^3 = \omega_2^1 \wedge \omega_1^3 \end{cases}$$

$\omega_1^2 = \Gamma_{1\alpha}^2 \theta^\alpha$

$$\begin{cases} (h_{11}\theta^1 + h_{12}\theta^2) = (\Gamma_{11}^2\theta^1 + \Gamma_{12}^2\theta^2) \wedge (h_{21}\theta^1 + h_{22}\theta^2) \\ (h_{21}\theta^1 + h_{22}\theta^2) = -(\Gamma_{11}^2\theta^1 + \Gamma_{12}^2\theta^2) \wedge (h_{11}\theta^1 + h_{12}\theta^2) \end{cases}$$

$$\begin{aligned} (\nabla_2 h_{11})\theta^2 \wedge \theta^1 - h_{11}\omega_2^1 \wedge \theta^2 + (\nabla_1 h_{12})\theta^1 \wedge \theta^2 - h_{12}\omega_1^2 \wedge \theta^1 &= (\Gamma_{11}^2 h_{22} - \Gamma_{12}^2 h_{21})\theta^1 \wedge \theta^2 \\ (\nabla_2 h_{21})\theta^2 \wedge \theta^1 - h_{21}\omega_2^1 \wedge \theta^2 + (\nabla_1 h_{22})\theta^1 \wedge \theta^2 - h_{22}\omega_1^2 \wedge \theta^1 &= -(\Gamma_{11}^2 h_{12} - \Gamma_{12}^2 h_{11})\theta^1 \wedge \theta^2 \end{aligned}$$

$$\begin{aligned} \nabla_2 h_{11} - \nabla_1 h_{12} &= \Gamma_{11}^2 h_{11} + \Gamma_{12}^2 h_{12} + \Gamma_{12}^2 h_{21} - \Gamma_{11}^2 h_{22} \\ \nabla_2 h_{21} - \nabla_1 h_{22} &= -\Gamma_{12}^2 h_{11} + \Gamma_{11}^2 h_{12} + \Gamma_{11}^2 h_{21} + \Gamma_{12}^2 h_{22} \end{aligned}$$

$$\omega_1^2 = \omega_1^\gamma \wedge \omega_\gamma^2 + \Omega_1^2$$

γ ω_1^1, ω_2^2

$$\omega_1^2 = \Omega_1^2$$

Ω_1^2 $-K\theta^1 \wedge \theta^2$ K $-2R^2_{121}$

$\omega_1^2 = \Omega_1^2$

$$\Gamma_{11}^2 = -\frac{(\sqrt{E})}{\sqrt{EG}}, \Gamma_{12}^2 = \frac{(\sqrt{G})}{\sqrt{EG}}$$

$$\begin{cases} \dfrac{\partial b_{11}}{\partial v} - \dfrac{\partial b_{12}}{\partial u} =& \tilde{\Gamma}_{12}^1 b_{11} + \tilde{\Gamma}_{12}^2 b_{21} - \tilde{\Gamma}_{11}^1 b_{12} - \tilde{\Gamma}_{11}^2 b_{22} \\ \dfrac{\partial b_{21}}{\partial v} - \dfrac{\partial b_{22}}{\partial u} =& \tilde{\Gamma}_{22}^1 b_{11} + \tilde{\Gamma}_{22}^2 b_{21} - \tilde{\Gamma}_{21}^1 b_{12} - \tilde{\Gamma}_{21}^2 b_{22} \end{cases}$$

$$\begin{aligned} \tilde{\Gamma}_{11}^1 =& \frac{1}{2}\frac{\partial \quad E}{\partial u}, \qquad \tilde{\Gamma}_{12}^1 = \tilde{\Gamma}_{21}^1 = \frac{1}{2}\frac{\partial \quad E}{\partial v}, \qquad \tilde{\Gamma}_{22}^1 = -\frac{1}{2E}\frac{\partial G}{\partial u} \\ \tilde{\Gamma}_{11}^2 =& -\frac{1}{2G}\frac{\partial E}{\partial v}, \qquad \tilde{\Gamma}_{12}^2 = \end{aligned}$$

$$\theta^1 = \quad \tilde{\phi}\tilde{\theta}^1 + \quad \tilde{\phi}\tilde{\theta}^2, \quad \theta^2 = - \quad \tilde{\phi}\tilde{\theta}^1 + \quad \tilde{\phi}\tilde{\theta}^2$$

$$\begin{aligned}\theta^1 &= - \quad \tilde{\phi} \ \tilde{\phi} \wedge \tilde{\theta}^1 - \quad \tilde{\phi}\tilde{\omega}_2^1 \wedge \tilde{\theta}^2 + \quad \tilde{\phi} \ \tilde{\phi} \wedge \tilde{\theta}^2 - \quad \tilde{\phi}\tilde{\omega}_1^2 \wedge \tilde{\theta}^1 \\ &= \ \tilde{\phi} \wedge (- \quad \tilde{\phi}\tilde{\theta}^1 + \quad \tilde{\phi}\tilde{\theta}^2) - \tilde{\omega}_2^1 \wedge (- \quad \tilde{\phi}\tilde{\theta}^1 + \quad \tilde{\phi}\tilde{\theta}^2) \\ &= -(- \ \tilde{\phi} + \tilde{\omega}_2^1) \wedge \theta^2\end{aligned}$$

$$\begin{aligned}\theta^2 &= - \quad \tilde{\phi} \ \tilde{\phi} \wedge \tilde{\theta}^1 + \quad \tilde{\phi}\tilde{\omega}_2^1 \wedge \tilde{\theta}^2 - \quad \tilde{\phi} \ \tilde{\phi} \wedge \tilde{\theta}^2 - \quad \tilde{\phi}\tilde{\omega}_1^2 \wedge \tilde{\theta}^1 \\ &= - \ \tilde{\phi} \wedge (\quad \tilde{\phi}\tilde{\theta}^1 + \quad \tilde{\phi}\tilde{\theta}^2) - \tilde{\omega}_1^2 \wedge (\quad \tilde{\phi}\tilde{\theta}^1 + \quad \tilde{\phi}\tilde{\theta}^2) \\ &= -(\ \tilde{\phi} + \tilde{\omega}_1^2) \wedge \theta^1\end{aligned}$$

$$\omega_1^2 = \ \tilde{\phi} + \tilde{\omega}_1^2$$

$$\kappa_{\mathfrak{g}} = \frac{(\ \tilde{\phi} + \tilde{\omega}_1^2)|_C}{s} = \frac{\tilde{\phi}}{s} + \frac{\tilde{\omega}_1^2|_C}{s}$$

$$\tilde{\phi} \equiv 0$$

$$\kappa_{\mathfrak{g}} \ s = \ \tilde{\phi} + \tilde{\omega}_1^2|_C$$

C

$$\oint_C \kappa_{\mathfrak{g}} \ s = \oint_C \ \tilde{\phi} + \oint_C \tilde{\omega}_1^2$$

$$\tilde{\omega}_1^2 = \tilde{\Omega}_1^2 = -K\tilde{\theta}^1 \wedge \tilde{\theta}^2$$

$$\oint_C \tilde{\omega}_1^2 = \iint_D \ \tilde{\omega}_1^2 = -\iint_D K \ \sigma$$

$\sigma = \tilde{\theta}^1 \wedge \tilde{\theta}^2$

$$\oint_C \kappa_{\mathfrak{g}} \ s + \iint_D K \ \sigma = \oint_C \ \tilde{\phi}$$

α C s

$$\oint_C \ \tilde{\phi} = 2\pi - \Sigma_{\ =1} \alpha$$

$$\oint_C \kappa_{\mathfrak{g}} \ s + \iint_D K \ \sigma = 2\pi - \Sigma_{\ =1} \alpha$$

S (u,v)

$E(\ u)^2 + G(\ v)^2$ C $u-$ ϕ

$$\kappa_{g} = \frac{\phi(s)}{s} - \frac{1}{2\sqrt{EG}}\left(E\ \frac{u}{s} - G\ \frac{v}{s}\right)$$

$\oint_C \frac{\phi(\ )}{}\ s$ $2\pi - \Sigma_{=1}\alpha$

$$\oint_C \frac{1}{2\sqrt{EG}}\left(E\ \frac{u}{s} - G\ \frac{v}{s}\right)\ s = \oint_C \frac{1}{2\sqrt{EG}}(E\ \ u - G\ \ v)$$

$$-\frac{1}{2}\iint_D \left(\frac{G}{\sqrt{EG}}\right)\ + \left(\frac{E}{\sqrt{EG}}\right)\ \ u\ v$$

$\sigma = \sqrt{EG}\ u\ v$

$$\iint_D -\frac{1}{\sqrt{EG}}\left(\left(\frac{\sqrt{E}}{\sqrt{G}}\right)\ + \left(\frac{\sqrt{G}}{\sqrt{E}}\right)\ \right)\ \sigma$$

$$K = -\frac{1}{\sqrt{EG}}\left(\left(\frac{\sqrt{E}}{\sqrt{G}}\right)\ + \left(\frac{\sqrt{G}}{\sqrt{E}}\right)\ \right)$$

$\iint_D K\ \sigma$

C D S

D

S

D

2.3 κ_g

$+a^2$ u κ_g

1. S $I=(\ u)^2+(u^2+a^2)(\ v)^2$ $C: u+v=0$

κ_g $\phi(s)$

$\tilde{\phi}$

$\{e_1, e_2\}$ e_1 C

$\{e_1, e_2\}$ C $\frac{\partial}{\partial} - \frac{\partial}{\partial}$

$$e_1 = \frac{1}{\sqrt{1+a^2+u^2}}\left(\frac{\partial}{\partial u} - \frac{\partial}{\partial v}\right)$$

e_1

$$e_2 = \frac{\sqrt{a^2+u^2}}{\sqrt{1+a^2+u^2}}\left(\frac{\partial}{\partial u} + \frac{1}{a^2+u^2}\frac{\partial}{\partial v}\right)$$

ω_1^2

I e_1, e_2

$$\theta^1 = \frac{1}{\sqrt{1+a^2+u^2}}\ u - \frac{a^2+u^2}{\sqrt{1+a^2+u^2}}\ v; \quad \theta^2 = \frac{\sqrt{a^2+u^2}}{\sqrt{1+a^2+u^2}}(\ u + \ v)$$

θ^1 θ^1

$$\theta^1 = -\left(\frac{2u}{\sqrt{1+a^2+u^2}} - \frac{u(a^2+u^2)}{(\sqrt{1+a^2+u^2})^3}\right)$$ 32333303[(221121233[(1)]1210333[(=)])]121032101

θ

$\theta TdfTd<beb<bcTTf\pi hD\partial\partial\alpha Dc\sigma$

∀∀∀a ±E∀·∀·∀∀≡∀∀∈× ≡ ·∧− ·∀·≡{−

$$\omega_1^2|_C = -\omega_2^1|_C = -\frac{u(2+a^2+u^2)}{\sqrt{a^2+u^2}(1+a^2+u^2)}\ \ u$$

$$s = \sqrt{1+a^2+u^2}\ \ u$$

$$\kappa_{\mathfrak{g}} = -\frac{u(2+a^2+u^2)}{\sqrt{a^2+u^2}(\sqrt{1+a^2+u^2})^3}$$

3

3.1

$\tilde{e}_1, \tilde{e}_2$ $\tilde{\theta}^1, \tilde{\theta}^2$ C $\tilde{e}_1$ $\tilde{\phi}$

$\tilde{\theta}^1, \tilde{\theta}^2$

$$\kappa_{\mathfrak{g}} = \frac{\tilde{\phi}}{s} + \frac{\tilde{\omega}_1^2|_C}{s}$$

$$I = E(\ \ u)^2 + G(\ \ v)^2$$

$$\tilde{\theta}^1 = \sqrt{E}\ \ u, \quad \tilde{\theta}^2 = \sqrt{G}\ \ v$$

$$\tilde{\omega}_1^2 = -\frac{(\sqrt{E})}{\sqrt{G}}\ \ u + \frac{(\sqrt{G})}{\sqrt{E}}\ \ v$$

$\tilde{\phi} = \phi$

$$\kappa_{\mathfrak{g}} = \frac{\phi(s)}{s} - \frac{1}{2\sqrt{EG}}\left(E\ \ \frac{u}{s} - G\ \ \frac{v}{s}\right)$$

e_1 90°

e_2 $\tilde{\phi} \equiv 0$

$$\kappa_{\mathfrak{g}} = \frac{\omega_1^2|_C}{s}$$

$\{e_1, e_2, \vec{n}\}$

3.2

S

$\{e_1, e_2, \vec{n}\}$ e_1, e_2 $\theta^1, \theta^2, \theta^3$ $\theta^3|\ \equiv 0$

S C $\{e_1, e_2\}$ e_1 C e_2

e_1 90° θ^1, θ^2 $\theta^2|_C \equiv 0$

$\theta^1, \cdots, \theta$

$\theta^{\ +1}, \cdots, \theta$ $\theta^{\ +1} \equiv \cdots \equiv \theta \ \equiv 0$

A 1

θ^1 θ^2

$$
\begin{aligned}
\theta^2 &= \left(\frac{u}{\sqrt{a^2+u^2}\sqrt{1+a^2+u^2}} - \frac{u\sqrt{a^2+u^2}}{(\sqrt{1+a^2+u^2})^3} \right) \ u \wedge \ v \\
&= \frac{u}{\sqrt{a^2+u^2}(\sqrt{1+a^2+u^2})^3} \ u \wedge \ v \\
&= -\frac{u}{(\sqrt{a^2+u^2})^3(1+a^2+u^2)} \ u \wedge \left(\frac{1}{\sqrt{1+a^2+u^2}} \ u - \frac{a^2+u^2}{\sqrt{1+a^2+u^2}} \ v \right) \\
&= -\frac{u}{(\sqrt{a^2+u^2})^3(1+a^2+u^2)} \ u \wedge \theta^1
\end{aligned}
$$

$$
\omega_1^2 = \frac{u}{(\sqrt{a^2+u^2})^3(1+a^2+u^2)} \ u + d(u,v)\theta^1
$$

$\omega_2^1 + \omega_1^2 = 0$

$$
\frac{u}{(\sqrt{a^2+u^2})^3(1+a^2+u^2)} \left((a^2+u^2)^2 + 2(a^2+u^2) + 1 \right) \ u + c(u,v)\theta^2 + d(u,v)\theta^1 = 0
$$

$$
-\frac{u(1+a^2+u^2)}{(\sqrt{a^2+u^2})^3} \ u = c(u,v)\theta^2 + d(u,v)\theta^1
$$

$$
\begin{cases}
c(u.v) = -\dfrac{u\sqrt{1+a^2+u^2}}{a^2+u^2} \\
d(u,v) = -\dfrac{u\sqrt{1+a^2+u^2}}{(\sqrt{a^2+u^2})^3}
\end{cases}
$$

$$
\omega_2^1 = \frac{u}{\sqrt{a^2+u^2}} \left(\frac{u}{1+a^2+u^2} - \ v \right) = -\omega_1^2
$$